Degradation of Power Combining Efficiency Due to Variability Among Signal Sources

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Abstract—**The power combining efficiency of a symmetric n-way power combiner depends on the degree of imbalance among its input signals. This paper establishes the worst-case efficiency for a combiner when its input signal amplitudes and phases are uncertain, but constrained to given ranges. This result is then used to deduce the permissible tolerance in the uniformity of components used in power combiner construction, given the maximum acceptable efficiency degradation.**

I. Introduction

Consider a linear n-way power combiner, excited at its input ports by n independent (i.e., uncoupled) one-port linear source networks. The combining efficiency η_c of the power combiner is a measure of the extent to which the output power P_o of the combiner approaches the arithmetic sum of the powers P_{av}, available from each of the n individual signal sources being combined; it is defined by

$$P_o = \eta_c \sum_{k=1}^{n} P_{av,k} \tag{1}$$

when P_o has been maximized by impedance matching each of the n sources and the load connected to the combiner ports, so that the value of η_c is truly a figure of merit of the combiner. It is well known that, for a combiner with n-way symmetry, the combining efficiency is the highest when the incoming signals are identical with each other in amplitude and phase. This maximum efficiency, to be denoted hereafter by η_{max}, is an intrinsic property of the combiner, since it is determined solely by the $[S]$ parameters of the combiner, and is limited only by losses in the combiner.

When the signals are not identical in amplitude and phase, they must be added vectorially (as a consequence of the linearity of the combiner), so that the summation in (1) can be written [1] as

$$P_o = \eta_{max} \frac{1}{n}\left[\left(\sum_{k=1}^{n} \sqrt{P_{av,k}} \cos\theta_k\right)^2 + \left(\sum_{k=1}^{n} \sqrt{P_{av,k}} \sin\theta_k\right)^2\right] \tag{2a}$$

where $P_{av,k}$ and θ_k are the available power and the phase angle (with respect to some arbitrary reference) of the kth input signal. The corresponding efficiency of combining is then found from (1) and (2a), and is given by

$$\frac{\eta_c}{\eta_{max}} = \frac{\left[\left(\sum_{k=1}^{n} \sqrt{P_{av,k}} \cos\theta_k\right)^2 + \left(\sum_{k=1}^{n} \sqrt{P_{av,k}} \sin\theta_k\right)^2\right]}{\left(n \sum_{k=1}^{n} P_{av,k}\right)}. \tag{2b}$$

This efficiency is not an intrinsic property of the combiner (since it depends not only on the combiner $[S]$ parameters but also the input signals) and is less than the intrinsic combiner efficiency (since the ratio in (2b) is necessarily less than unity).

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Equation (2) is not useful by itself for design purposes, since it requires a knowledge of the amplitudes and phases of all n input signals. A more useful result would be an expression for the reduction in output power and efficiency as a function of the degree of imbalance among the input signals. The purpose of this paper is to deduce such an expression.

II. Previously Known Results

Several expressions and charts are available in the literature [2]–[9] for determining the combiner output power and efficiency degradation due to input signal imbalance. Their basis and applicability are best demonstrated by showing how each follows from (2) as a special case:

(i) Identical-Phases, Unequal-Amplitudes Case: If in (2) all θ_k are identical, and if m out of the n input signals have a reduced power level rP_{av} while the remaining $n - m$ signals have the same full power level P_{av}, the reduction in the power output and the efficiency are found from (2) as

$$\frac{P_o}{nP_{av}\eta_{max}} = \left[1 - \frac{m}{n}\left(1 - \sqrt{r}\right)\right]^2 \tag{3a}$$

$$\frac{\eta_c}{\eta_{max}} = \frac{\left[1 - \frac{m}{n}\left(1 - \sqrt{r}\right)\right]^2}{1 - \frac{m}{n}(1 - r)}. \tag{3b}$$

These results were given in [2], and have been repeated by others, sometimes for the limiting case in which $r = 0$, i.e., where the m sources fail entirely [3], [4].

(ii) Identical-Amplitudes, Unequal-Phases Case: If all $P_{av,k}$ are identical, and if m out of the n signals are out of phase with respect to the remaining $n - m$ signals (all of which are in phase with each other) by the same angle ϕ, the reduction in power output and efficiency are found from (2) to be [2]

$$\frac{P_o}{nP_{av}\eta_{max}} = \frac{\eta_c}{\eta_{max}} = 1 - 2\left(\frac{m}{n}\right)\left(1 - \frac{m}{n}\right)(1 - \cos\phi). \tag{4}$$

(iii) Two-Input Case: If $n = 2$, and one input signal has both a reduced amplitude (by a factor r) and a phase shift (by angle ϕ) with respect to the other, the resulting reductions in the output power and efficiency are given by

$$\frac{P_o}{2P_{av}\eta_{max}} = \frac{1}{4}(1 + r + 2\sqrt{r}\cos\phi) \tag{5a}$$

$$\frac{\eta_c}{\eta_{max}} = \frac{1}{2} + \frac{\sqrt{r}}{r + 1}\cos\phi. \tag{5b}$$

These results were deduced in [5], and have been repeated by others [6], [7]. Since they contain only two independent variables r and ϕ, these equations have also been presented in the literature as nomograms [5]–[8].

Equations (3) to (5) express the degradation of P_o and η_c in terms of m, r, and ϕ, which are measures of the degree of imbalance among the input signals. These expressions have the following limitations:

(a) Equations (3) and (4) assume that the m signal sources deviate from the norm either only in amplitude or only in phase. Equation (5) permits both deviations, but is limited to two inputs.

(b) They assume that the $n - m$ signal sources have been matched with each other, as would be typical in the hybrid construction where each input port of the combiner is individually tuned.

(c) They assume that the n-way combiner has a perfect n-way symmetry, with no parameter variation from port to port.

These expressions are therefore useful for studying the combiner performance deterioration due to a failure of sources supplying the input signals.

The purpose of this paper is to present an alternative expression for estimating the reduction in η_c of the combiner, as a function of the range of scatter in the input signal amplitudes and phases. This expression will permit the amplitudes and phases of all input signals to be arbitrary within some specificed ranges, as would be typical in monolithic fabrication where the signals are supplied by a set of amplifier or oscillator chips that fall within the specification window. A scatter in the combiner parameters can also be accounted for in this expression (i.e., a perfect n-way symmetry of the combiner is not assumed). The expression will therefore be useful for studying the deterioration of combiner performance with component tolerance, production variability, and the gradual degradation of all sources due to aging.

III. The Worst-Case Efficiency

From (1), the combining efficiency is given by

$$\eta_c \equiv \left. \frac{P_o}{\sum P_{\text{av},k}} \right|_{\Gamma_L = \Gamma_{s,k} = 0, k = 1, 2, \cdots, n} \equiv \frac{\left| \sum S_{ok} b_{g,k} \right|^2}{\sum |b_{g,k}|^2} \tag{6}$$

where

- $b_{g,k}$ = the complex amplitude of the power wave that would be launched on a transmission line of characteristic impedance R_o by the source connected at the kth input port,
- Γ_L, $\Gamma_{s,k}$ = reflection coefficients of the terminations at the load and source ports, defined with reference to R_o.
- S_{ok} = an element of the scattering matrix of the $(n + 1)$ port combiner, defined with reference to R_o, representing the transmission from kth input port to the output port "o,"

and the summation (in this and all subsequent expressions) extends over the range $k = 1$ to n. The goal is to determine the worst-case (i.e., the minimum) value of η_c, minimized with respect to the amplitudes and phases of the signals to be combined, subject to the constraint that their scatter is restricted to a known range.

The results can be written more compactly by defining the following variables for the amplitude and phase:

$$B_k = |b_{g,k}| \tag{7a}$$

$$C_k = |S_{ok}| \tag{7b}$$

$$\rho_k = \text{Arg}\,(b_{g,k} S_{ok}). \tag{7c}$$

The combining efficiency η_c in (6) can be expressed as a function of these $3n$ real variables as follows:

$$\eta_c = \frac{\left| \sum B_k C_k \exp\,(j\rho_k) \right|^2}{\sum B_k^2}. \tag{8}$$

The scatter in the values of the three variables B_k, C_k, and ρ_k from port to port is responsible for lowering the value of η_c. In a typical production situation where the component tolerances are specified, the scatter in these variables will be restricted to some range, which can be defined as follows:

$$b_{\min} \leq |b_{g,k}| \leq M_b b_{\min} \tag{9a}$$

$$s_{\min} \leq |S_{ok}| \leq M_s s_{\min} \tag{9b}$$

$$\rho_{\min} \leq \rho_k \leq \rho_{\min} + 2\delta_{\max}. \tag{9c}$$

Since the efficiency is influenced not by the total phase shift ρ_k, but by its deviations from say the center of the range of scatter,

$$\delta_k \equiv \rho_k - (\rho_{\min} + \delta_{\max}) \tag{9d}$$

the range in (9c) can be rewritten as

$$-\delta_{\max} \leq \delta_k \leq \delta_{\max}. \tag{9e}$$

Subject to the constraints in (9), the lowest value of η_c in (8) is bounded as follows:

$$\eta_c \geq \frac{4 M_s M_b \cos^2 \delta_{\max}}{(1 + M_s M_b)^2} \sum |S_{ok}|^2. \tag{10}$$

A proof of this result is contained in the Appendix. The right-hand side of (10) is the worst-case value of η_c, and is the principal result of this paper.

This result can also be expressed in terms of the maximum efficiency. The $\eta_{\max}$ can be identified by making the n input signals $b_{g,k}$ identical with each other in (6):

$$\eta_{\max} = \sum |S_{ok}|^2 \tag{11}$$

so that the worst-case efficiency in (10) can be written as

$$\frac{\text{Min}\,[\eta_c]}{\eta_{\max}} = \frac{4 M_s M_b \cos^2 \delta_{\max}}{(1 + M_s M_b)^2}. \tag{12}$$

Some appreciation for the tightness of the lower-bound on η_c can be developed by comparing it with the actual η_c for those special cases in which the combining efficiency can indeed be calculated with the help of the previously known results listed in Section II. For example, for a symmetrical combiner (i.e., $M_s = 1$) with inputs having identical amplitudes (i.e., $M_b = 1$) and two unequal phases (i.e., $2\delta_{\max} = \phi$), the *worst-case* efficiency in (12) is

$$\frac{\text{Min}\,[\eta_c]}{\eta_{\max}} = \cos^2(\phi/2) \tag{13}$$

This is the same as the *actual* efficiency calculated from (4) when $m = n/2$. For a two-way symmetrical combiner (i.e., $M_s = 1$) in which the input signal amplitudes have a ratio r (i.e., $M_b = 1/r$) and a phase difference ϕ (i.e., $2\delta_{\max} = \phi$), the worst-case combining efficiency is given by

$$\frac{\text{Min}\,[\eta_c]}{\eta_{\max}} = \frac{2r(1 + \cos\phi)}{(1 + r)^2}. \tag{14}$$

A comparison with the actual efficiency in (5b) shows that Min $[\eta_c]$ is within 2% of η_c for $r > 0.75$, within 10% of η_c for $r > 0.52$, and deviates increasingly from η_c as r becomes smaller.

IV. Applications

The above result can be utilized in several ways, such as (i) for determining the worst-case combiner performance, given the range of scatter in combiner parameters and signal sources, (ii) for calculating the maximum permissible tolerance in combiner and source

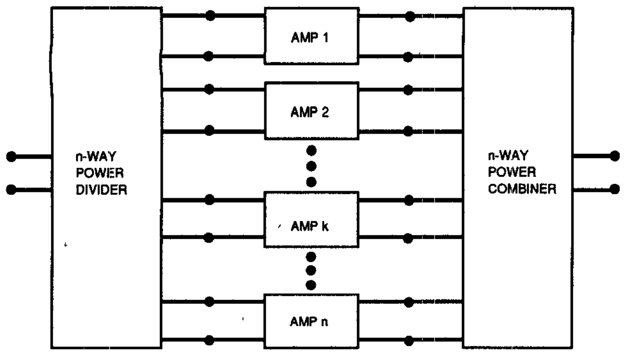

Fig. 1. A high-power amplifier based on n independent amplifiers, and n-way power divider and combiner.

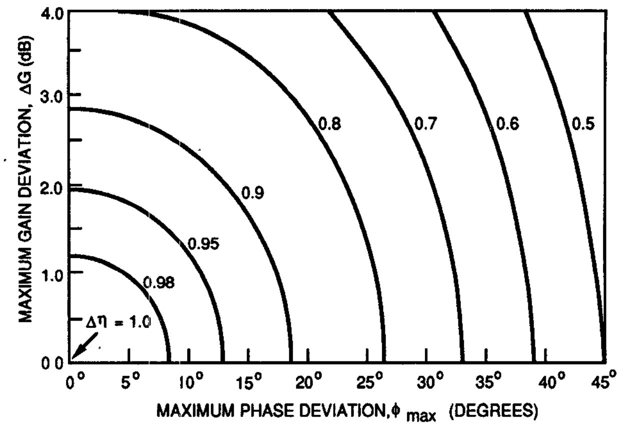

Fig. 2. Worst-case degradation in combining efficiency as a function of the maximum permissible gain variation (in dB), and the maximum permissible phase deviation (in degrees), among the n amplifiers combined together with perfect n-way power divider and combiner.

specifications that will guarantee that the combiner will meet a given efficiency requirement, and (iii) for cost—performance tradeoff, given the relationship between the cost and the tightness of specifications for components like source devices. As an illustrative example of the use of (10), the acceptable level of component tolerance will be deduced with its help for a typical high-power amplifier composed of an n-way power divider, n individual power amplifiers, followed by a power combiner as shown in Fig. 1. The n amplifiers, although nominally identical, will differ from each other due to fabrication tolerances, and will have among them some scatter in the values of their power gain and phase shift. This scatter must be kept within some limits so that the combining efficiency does not fall below the minimum acceptable value. The problem then is to determine the tolerance specification for the gain and phase shift of the n amplifiers to ensure that the combining efficiency remains acceptable.

In most cases, the scatter in the gain and phase-shift of the n amplifiers will be the dominant cause of signal imbalance in the high-power amplifier of Fig. 1, while the asymmetry caused by the power divider and combiner will be small and negligible. Under these conditions, $M_s = 1$ is a reasonable approximation, and the variability among $b_{g,k}$ at the combiner input ports is entirely due to the gain and phase-shift variability among the individual amplifiers. Under these conditions, (12) shows that the worst-case degradation in combining efficiency, to be denoted by $\Delta\eta$, is given by

$$\Delta\eta \equiv \frac{\text{Min }[\eta_c]}{\eta_{\max}} = \frac{4M_b \cos^2 \delta_{\max}}{(1 + M_b)^2}. \tag{15}$$

The parameters on the right-hand side of (15) depend on the permitted variability among amplifiers. Suppose the amplifiers are selected from a production lot by screening, and are considered acceptable provided that (i) their available power gain lies within $\pm\Delta G$ dB of its nominal value G_o, and (ii) their phase shift lies within $\pm\phi_{\max}$ degrees of its nominal value. The available power gain G_k of the kth amplifier, expressed in numeric units, lies in the interval

$$10^{(G_o - \Delta G)/10} \leq G_k \leq 10^{(G_o + \Delta G)/10} \tag{16}$$

It follows from the definitions in (9) that

$$M_b = 10^{\Delta G/10} \tag{17a}$$

$$\delta_{\max} = \phi_{\max}. \tag{17b}$$

The substitution of (17) in (15) then determines the worst-case combining efficiency degradation for the amplifier screening criterion employed. Fig. 2 shows contours of constant $\Delta\eta$ in the $\Delta G - \phi_{\max}$ plane, and can be used to select the gain and phase tolerances ΔG and $\phi_{\max}$ for a desired minimum combining efficiency. The contours also show that the tolerance on either the gain or the phase shift can be relaxed if the other is made more stringent, and thus allow for some trade-off in establishing the window of acceptability for the amplifiers. As a rule of thumb, a change of phase shift tolerance by one degree causes the same degradation in combining efficiency as a change of gain tolerance by approximately 0.15 dB. This observation agrees with the published results [5] for the case of a two-way combiner, which is the only case available in the literature for comparison.

Appendix
Lower Bound on Efficiency

The lower bound on η_c will be established by separating it into two parts, and then establishing a lower bound on each. For this purpose, η_c in (8) can be written as

$$\eta_c = \eta_a \eta_b \tag{A1}$$

where

$$\eta_a = \frac{\left|\sum B_k C_k \exp(j\delta_k)\right|^2}{\left(\sum B_k C_k\right)^2} \tag{A2}$$

and

$$\eta_b = \frac{\left(\sum B_k C_k\right)^2}{\sum B_k^2}. \tag{A3}$$

The first part can be bounded as follows. Since the magnitude of a complex number cannot be less than its real part,

$$\eta_a \geq \frac{\left(\sum B_k C_k \cos \delta_k\right)^2}{\left(\sum B_k C_k\right)^2} \tag{A4}$$

$$\geq \frac{\left(\sum B_k C_k \cos \delta_{\max}\right)^2}{\left(\sum B_k C_k\right)^2} \tag{A5}$$

$$= \cos^2 \delta_{\max}. \tag{A6}$$

The inequality in (A5) assumes that the maximum phase deviation δ_{max} is no larger than $\pi/2$ radians, so that the function cos δ_k can be treated as a monotonic function of its argument. This is a reasonable assumption in the practical situation, since signals with larger phase deviations will cancel rather than add together, and are obviously unsuitable for combining with each other.

A lower bound for the second part η_b can be found from inequalities complimentary to Cauchy's inequality [8] as follows. It follows from (9a) and (9b) that

$$\frac{1}{M_b} \cdot \frac{s_{min}}{b_{min}} \le \frac{C_k}{B_k} \le M_s \frac{s_{min}}{b_{min}}. \quad \text{(A7)}$$

Consequently, each of the three factors in the product below is positive:

$$B_k^2 \left(\frac{C_k}{B_k} - \frac{s_{min}}{M_b b_{min}}\right)\left(\frac{M_s s_{min}}{b_{min}} - \frac{C_k}{B_k}\right) \ge 0. \quad \text{(A8)}$$

Multiplying out the three factors, and then summing over n such products, yields

$$\frac{s_{min}}{b_{min}}\left(M_s + \frac{1}{M_b}\right) \sum B_k C_k \ge \sum C_k^2 + \left(\frac{s_{min}}{b_{min}}\right)^2 \frac{M_s}{M_b} \sum B_k^2. \quad \text{(A9)}$$

If the right-hand side of (A9) is treated as the arithmetic mean of two quantities, which itself cannot be less than their geometric mean, it can be replaced by the geometric mean of those quantities; this leads to

$$\frac{s_{min}}{b_{min}}\left(M_s + \frac{1}{M_b}\right) \sum B_k C_k \ge 2\left[\sum C_k^2\right]^{1/2} \left[\left(\frac{s_{min}}{b_{min}}\right)^2 \frac{M_s}{M_b} \sum B_k^2\right]^{1/2}. \quad \text{(A10)}$$

After the two sides of (A10) are squared and simplified, the final result is

$$\eta_b \equiv \frac{\left(\sum B_k C_k\right)^2}{\sum B_k^2} \ge \frac{4 M_s M_b}{(1 + M_s M_b)^2} \sum C_k^2. \quad \text{(A11)}$$

Substitution of (A6) and (A11) in (A1) yields the lower bound for the combining efficiency given in (10).

It will now be shown that the combining efficiency can actually attain a value equal to the lower bound found above under one very special set of circumstances. Consider the special case where the forward transfer functions S_{ok} of the combiner at $nM_b/(M_s + M_b)$ out of the n ports have the maximum permissible magnitude, while the signals at these ports come from sources with the minimum permissible magnitude of $b_{g,k}$; i.e.,

$$|S_{ok}| = M_s s_{min}, \quad |b_{g,k}| = b_{min}. \quad \text{(A12)}$$

Furthermore, half of these signals have a phase deviation δ_k equal to the maximum positive value, while the other half have a phase deviation equal to the maximum negative value*, i.e.,

$$\delta_k = \delta_{max} \quad \text{and} \quad \delta_k = -\delta_{max}. \quad \text{(A13)}$$

At the remaining $nM_s/(M_s + M_b)$ of the ports, the forward transfer functions of the combiner, S_{ok} are at the minimum, while the signals have the maximum permissible strengths:

$$|S_{ok}| = s_{min}; \quad |b_{g,k}| = M_b b_{min} \quad \text{(A14)}$$

with the phase deviation being δ_{max} for half of these ports and $-\delta_{max}$ for the other half. Substitution of these values for the variables in (6) shows that

$$\eta_c = \frac{\left|\frac{1}{2}\left(\frac{nM_b}{M_s + M_b}\right) M_s s_{min} b_{min} (e^{j\delta_{max}} + e^{-j\delta_{max}}) + \frac{1}{2}\left(\frac{nM_s}{M_s + M_b}\right) s_{min} M_b b_{min} (e^{j\delta_{max}} + e^{-j\delta_{max}})\right|^2}{\left(\frac{nM_b}{M_s + M_b}\right) b_{min}^2 + \left(\frac{nM_s}{M_s + M_b}\right) M_b^2 b_{min}^2} \quad \text{(A15)}$$

$$= \frac{4 M_s M_b}{(1 + M_s M_b)^2} \cos^2 \delta_{max} \left[\frac{nM_b}{M_s + M_b} M_s^2 s_{min}^2 + \frac{nM_s}{M_s + M_b} s_{min}^2\right]. \quad \text{(A16)}$$

This demonstrates that, for the special case considered, the combining efficiency attains a value equal to the lower bound in (10). Since the lower bound can actually be reached, two conclusions can be drawn: (i) The established lower bound is the tightest possible, and cannot be improved unless more severe constraints than in (9) are imposed; (ii) The lower bound is itself the minimum value, or the worst-case value, of the combining efficiency.

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*It is clear that the special conditions are exactly met only if the quantities $nM_s/(M_s + M_b)$ and $nM_b/(M_s + M_b)$ are even integers; if not, the equality in (10) does not hold.